APPROXIMATE EQUATION FOR THE MASS OR HEAT FLUX TO DROPLETS OF A CONCENTRATED CLOUD

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An approximate equation is derived for the Sherwood number, which is a measure of the convective diffusion to a liquid droplet under conditions of constrained flow.

The problems of convective diffusion to solid particles and liquid droplets (or gas bubbles) were studied in [1, 2] for the cases of high densities of these particles. The approach used there was to reduce the convective-diffusion equation to an equation of the heat-conduction type with a variable thermal conductivity; this approach was originally worked out in [3] (see also [4]). The constrained flow of an incompressible liquid around a concentrated system of particles was studied in [5, 6].

It follows from [6] that the stream function near a spherical interface can be written

$$\psi = -(U_1 a\xi + 3/4\, U_2 \xi^2)\sin^2\theta, \tag{1}$$

where the coefficients U_1 and U_2 are equal to the velocity at the equator ($\theta = 1/2\pi$) and to the relative flow velocity far from the particle, respectively, in the particular case of an isolated particle.

In the constrained flow around a solid sphere, a liquid droplet, or a gas bubble, in each case with the surface of the particle retarded by surfactants, the first term in (1) drops out, and the coefficient U_2 becomes, for a monodisperse system of particles of radius a [5, 6],

$$U_2 = \varphi(\rho)\, U,\ \ \varphi(\rho) = \frac{1}{2-3\rho}\left\{\left[18\rho\left(1-\frac{3}{2}\rho\right) + 81/4\rho^2\right]^{1/2} + 9/2\rho\right\} + 1. \tag{2}$$

In the flow around droplets or bubbles of viscosity μ', in the absence of surfactants, the second term in (1) is negligible in comparison with the first; in this case, according to [6], we can write

$$U_1 = 1/2\,\vartheta(\rho,\varkappa)\, U,\ \ \vartheta(\rho,\varkappa) = \frac{1+\zeta}{1+1/3\,\zeta+\varkappa},\quad \varkappa = \mu'/\mu, \tag{3}$$

where $\zeta = \zeta(\rho, \varkappa)$ is the positive root of a cubic equation discussed in [2]. Criterial relations between the Sherwood number and the Peclet number for a solid sphere (or a droplet with a retarded surface) and for a droplet or bubble with a free surface were also derived in [2]. In the derivation of the relation for a droplet or bubble with a free surface it was actually assumed that the stream function near the surface of the droplet or bubble can be approximated satisfactorily by the first term in (1). In flow around a droplet or bubble, however, the coefficients U_1 and U_2 are comparable in magnitude, so that the applicability of the criterial relations derived in [2] is restricted.

To derive a result valid for droplets of any viscosity, we use a modification of the von Kármán–Pohlhausen method, by analogy with [7-10].

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If both the inequalities $R = 2aU/\nu < 1$ and $P = 2aU/D \gg 1$ are satisfied, the flow around the particle can be analyzed in the Stokes approximation, and the transport of diffusing material to the surface of the particle can be analyzed in the approximation of a thin diffusion boundary layer.

The convective-diffusion equation is [1]

$$V_r \frac{\partial c}{\partial \xi} + V_\theta \frac{1}{a} \frac{\partial c}{\partial \theta} = D \frac{\partial^2 c}{\partial \xi^2}, \quad r = a + \xi \tag{4}$$

with the boundary conditions

$$c = c_0 \, (r \to \infty;\ r = a,\ \theta = 0), \quad c = 0 \, (r = a,\ \theta \neq 0), \tag{5}$$

where c_0 is the concentration of the material in the incoming flow. According to the self-consistent theory for the constrained flow around a cloud of particles [5, 6], the velocity V_θ can be written approximately as

$$V_\theta = \left[1 + \frac{\xi}{a}(M^* - 1)\right] U_1 \sin\theta, \quad M^* = M(\zeta - 1),$$
$$M = (3/2\, U/U_1 - 1). \tag{6}$$

The solution of the resulting boundary-value problem is precisely the same as that of the problem for an isolated particle if we replace the velocity U_0 in [8] at the surface of the droplet at $\theta = \pi/2$ by the velocity U_1 from (3) and replace the coefficient M by M*. As a result, we find the following equation for the thickness of the diffusion boundary layer:

$$y' = \frac{\frac{4}{P}\frac{1}{y}\frac{1}{\sin\theta} - \left[\frac{3}{5}\frac{U_1}{U} y + \frac{2}{15} y^2 \left(\frac{3}{2}(\zeta + 1) - \frac{U_1}{U}(\zeta + 2)\right)\right] \operatorname{ctg}\theta}{\frac{3}{10}\frac{U_1}{U} + \frac{2}{15} y \left(\frac{3}{2}(\zeta + 1) - \frac{U_1}{U}(\zeta + 2)\right)}, \tag{7}$$

where $y = \delta/a$. Introducing the average boundary-layer thickness δ_m, and assuming small values of the ratio U_1/U, specifically,

$$\frac{U_1}{U}\left[3/5 - 2/15 \frac{\delta_m}{a}(\zeta + 2)\right] \ll 1/5 \frac{\delta_m}{a}(\zeta + 1), \tag{8}$$

we find the following criterial relation from Eq. (7):

$$S = A P^{1/3}, \quad A = 1.035 (1 + \zeta)^{1/3}, \tag{9}$$

where $S = 2ak/D$ is the Sherwood number, and $k = D \int_0^\pi \frac{\sin\theta}{\delta} d\theta$ is the integral mass-transfer coefficient. For sufficiently large values of U_1/U,

$$3/10 \frac{U_1}{U} \gg 2/15 \frac{\delta_m}{a}\left[3/2(\zeta + 1) - \frac{U_1}{U}(\zeta + 2)\right] \tag{10}$$

we find, analogously,

$$S = B P^{1/2}, \quad B = 0.632 \vartheta^{1/2} (\rho, \varkappa). \tag{11}$$

Using the more exact calculation method of [3] (see also [1]), we find the coefficients

$$A = 0.998 (1 + \zeta)^{1/3}, \quad B = 0.65 \vartheta^{1/2} (\rho, \varkappa). \tag{12}$$

Using the same method as in [8], we can easily derive an interpolation dependence for the Sherwood and Peclet numbers which holds for intermediate cases:

$$S^3 - P\vartheta(\rho, \varkappa)[0.424 S + 0.331 (1 + 3\varkappa)] = 0. \tag{13}$$

Fig. 1a shows the Sherwood number as a function of the Peclet number according to (13) for various values of $\varkappa$ and a fixed value of ρ; Fig. 1b shows the Sherwood number as a function of the concentration ρ for various values of the Peclet number and for a fixed value of $\varkappa$. Shown for comparison (dashed curves) is the function $S = 0.65(\theta P)^{1/2}$. The interpolation dependence found here can be compared with the experimental data by the procedure of [2] (see also the discussion in [2]).

APPENDIX

Hydrodynamic Problem. We consider the liquid flow in a monodisperse cloud of small spherical droplets. Since the Reynolds numbers are small in this case, we can use the linearized hydromechanics equations in the Stokes form. We assume that the droplets are statistically independent and that there are no correlations among the spatial positions of the droplets. We assume the continuous suspension phase to be incompressible. We neglect the random fluctuations due to concentration fluctuations. Under these conditions we can use the "point-force" approximation proposed by Tam [5] for the solution of the problem of the steady-state flow in an array of solid particles; according to this approximation, the perturbations caused in the flow by the particles are replaced by the perturbations caused by point forces applied to the liquid at the centers of the particles.

In this situation the system of equations of motion is

$$\left.\begin{aligned} \Delta\bar{V} - \alpha^2\bar{V} &= 1/\mu\nabla p \\ \nabla\bar{V} &= 0 \end{aligned}\right\} \text{outside the droplet,}$$
$$\left.\begin{aligned} \nabla\bar{V}' &= 1/\mu'\nabla p' \\ \nabla\bar{V}' &= 0 \end{aligned}\right\} \text{within the droplet,} \tag{1.A}$$

and the boundary conditions are

$$\text{at } r=a \begin{cases} V_n = V_n' = 0 & \text{(the normal component of the velocity vector vanishes)} \\ V_\theta = V_\theta' & \text{(the tangential components of the velocity vectors vanish)} \\ (\sigma\bar{n})_\theta = (\sigma'\bar{n})_\theta & \text{(the tangential stresses are equal).} \end{cases} \tag{2.A}$$

Here σ is the stress tensor; $\bar{V} \to \vec{U}$ in the limit $r \to \infty$; $\bar{V}'$, p' are bounded in the limit $r \to 0$; α is a coefficient incorporating the constraint on the flow (it depends on the particle concentration and the ratio of the viscosities of the continuous and disperse phases); and p and p' are the pressures in the continuous and disperse phases, respectively. Employing the method used previously in [11] to solve the problem of the laminar flow of a viscous liquid around a sphere, we find the solution of System (1.A), (2.A) for the velocities: for the continuous phase,

$$\begin{gathered} \bar{V} = [1 - E\exp(-x)(x^2+x+1)x^{-3} + Gx^{-3}]\bar{U} \\ + [E\exp(-x)(x^2+3x+3)x^{-3} - 3Gx^{-3}](\bar{U}\bar{n})\bar{n}, \\ E = \frac{1}{2}\zeta\Lambda\exp\zeta,\ G = \frac{1}{2}\zeta[\zeta^2 + (\zeta+1)\Lambda],\ x = \zeta r', \\ \Lambda = \frac{2+3\varkappa}{1+\zeta/3+\varkappa},\quad \zeta = \alpha a,\ r' = r/a; \end{gathered} \tag{3.A}$$

for the disperse phase

$$\begin{gathered} \bar{V}' = \left(E'x^2 + \frac{2}{3}G'\right)\bar{U} - \frac{1}{2}E'x^2(\bar{U}\bar{n})\bar{n}, \\ E' = \frac{1}{\zeta^2}\vartheta,\ G' = 3/4\,\vartheta,\ \vartheta = \frac{1+\zeta}{1+\zeta/3+\varkappa}. \end{gathered} \tag{4.A}$$

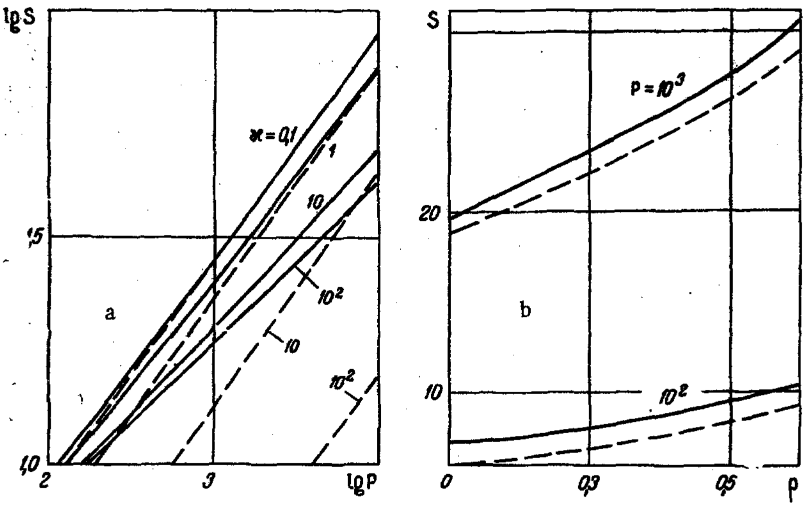

Fig. 1. Sherwood number S in Eq. (13). a) As a function of P for various values of ϰ (the curve labels) and a fixed value of ρ (ρ = 0.4); b) as a function of ϰ for various values of P (the curve labels) and a fixed value of ϰ(ϰ = 1). Solid curve) Eq. (13); dashed) $S = 0.65(P)^{1/2}$.

In the limit $\mu \to \infty$, we find from (3.A) the solution for an ensemble of hard spheres [5]. Expanding the projection of the velocity on the θ direction in a series in powers of $\xi (r = a + \xi)$, and retaining only the first term in the series for small ξ, we find, approximately,

$$V_\theta = \left[1 + (1 + 3\varkappa)\frac{\xi}{a}\right] U_1 \sin\theta. \tag{5.A}$$

The stream function ψ is calculated in the standard manner.

NOTATION

a, radius of particle; c, concentration of the material; $R = 2aU/\nu$, Reynolds number; $S = 2ak/D$, Sherwood number; $P = 2aU/D$, Peclet number; U, filtration velocity; V_θ, tangential velocity of liquid; θ, angular variable; (ρ, ϑ) function defined in (3); $\varkappa = \mu'/\mu$; μ, viscosity of continuous medium; μ', viscosity of particle material; ξ, radial distance from the surface of the particle; ν, kinematic viscosity of surrounding medium; D, diffusion coefficient; $y = \delta/a$; δ, thickness of diffusion boundary layer; δ_m, average thickness of diffusion boundary layer; M and M*, coefficients in Eq. (6); A, coefficient in (9); B, coefficient in (11); k, integral mass-transfer coefficient; U_1 and U_2, coefficients in Eq. (1); ρ, concentration of particles; $\zeta = \zeta(\rho, \vartheta)$ function defined in (3); ψ, stream function; φ, function defined in (2); U_0, value of V_θ at the surface of the particle at $\theta = \pi/2$.

LITERATURE CITED

1. Yu. A. Buevich, Inzh.-Fiz. Zh., 23, 709 (1972).
2. Yu. A. Buevich and Yu. A. Korneev, Inzh.-Fiz. Zh., 25, 594 (1973).
3. V. G. Levich, Physicochemical Hydrodynamics, Prentice-Hall, Englewood Cliffs, New Jersey (1962).
4. S. K. Friedlander, AIChE J., 7, 347 (1961).
5. C. K. W. Tam, J. Fluid Mech., 38, 537 (1969).
6. Yu. A. Buevich and V. G. Markov, Prikl. Mat. Mekh., 36, 480 (1972).
7. G. A. Aksel'rud, Inzh.-Fiz. Zh., 27, 1 (1953).
8. E. Rukenstein, Chem. Eng. Sci., 19, 131 (1964).
9. I. Yaron and B. Gal-Or, Int. J. Heat Mass Transfer, 14, 727 (1971).
10. R. Pfeffer, Ind. Eng. Chem. Func., 3, 380 (1964).
11. L. D. Landau and E. M. Lifshits (Lifshitz), Fluid Mechanics, Addison-Wesley, Reading, Massachusetts (1959).